Landau Fermi Liquid Theory for Collective Modes in 3D Dirac Materials

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Outline and Motivation

- Introduction to Fermi Liquid Theory
- A Graphene Break
- Local Fermi Liquid
- A Microscopic Model for a Dirac Liquid
- Consequences for the Fermi Velocity and Collective Modes
- Summary and a Question, What if,

$$v_f^0 = c?$$

Landau Fermi Liquid Theory

Landau's "Aha!" Moment: For low T properties of "strongly interacting" Fermi gases/liquids, e.g., cold atom (Fermi) gases, ^{3}He, metals, and nuclear/neutron matter, are qualitatively the same as a free Fermi gas!

The Entropy density, S (or specific heat C_V): $S = \gamma T$

where, $\gamma = \frac{\pi^2}{2} N(0)$ and $N(0) = \frac{3n}{2T_F}$

For $T << T_F$, the spin susceptibility and compressibility are independent of T, as in the free Fermi gas:

$$\chi = \frac{N(0)}{1 + F_0^a}, \quad \text{and} \quad (n^2\kappa)^{-1} = \frac{1 + F_0^s}{N(0)} = \frac{\partial \mu}{\partial n}$$

Lets look at the Energy and Density Range of Fermi Liquids

- The Fermi liquids Landau's "bold conjecture" covers a broad range of orders of magnitude for energy and density:

  - Nuclear Matter: $\varepsilon_F^{nm} = 37 MeV$

    $n_{nm} = 0.17\, fm^{-3} = 0.17 \times 10^{15} A^{-3}$

  - ^{6}Li – gas: $\varepsilon_F^{Li} = 2.2 \times 10^{-8} meV$

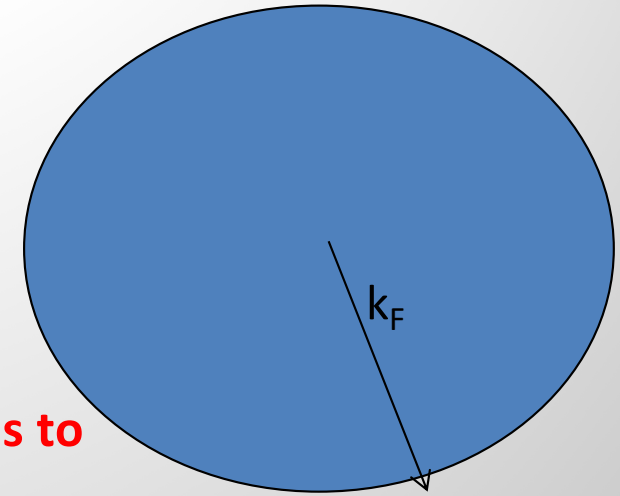
    $n_{Li} = 0.16 \times 10^{-10} A^{-3}$

There are 18 orders of magnitude in energy and 25 orders of magnitude in density!

One-To-One Correspondence

1) The volume of the Fermi sphere is the same for the interacting and non-interacting system:

$$n = \sum_{p\sigma} n^{(0)}_{p\sigma} = \sum_{p\sigma} n^{0}_{p\sigma} = \frac{k_F^3}{3\pi^2}$$



2) Each energy level in the interacting system corresponds to one and only one energy level in the non-interacting system:

$$\varepsilon^{(0)}_{p\sigma} \rightarrow \varepsilon^{0}_{p\sigma}$$

and, $\varepsilon^{(0)}_{p\sigma}$, is the quasiparticle Hamiltonian.

Basic assumption of LFLT is one-to one correspondence.

Why? Think Entropy!

Landau Fermi Liquid Theory Continued

Fluctuations in the Energy Density: $\delta E/V = (E - E_0)/V$

$$\delta E/V = \sum_{p\sigma} \varepsilon_{p\sigma} \delta n_{p\sigma} + \tfrac{1}{2} \sum_{p\sigma, p'\sigma'} f_{pp'}^{\sigma\sigma'} \delta n_{p\sigma} \delta n_{p'\sigma'} + \ldots$$

$$\sum_{p\sigma} \delta n_{p\sigma} = \delta n = n(r,t) - n_0 \quad \text{(Fluctuations of the density)}$$

$$\sum_{p\sigma} \sigma \delta n_{p\sigma} = \delta m = m(r,t) - m_0 \quad \text{(Fluctuations of the magnetization density)}$$

with the interactions in a spin-rotation invariant system given by,

$$f_{pp'}^{\sigma\sigma'} = f_{pp'}^{s} + f_{pp'}^{a} \sigma \cdot \sigma'$$

and the Landau parameters:

$$N(0) f_{pp'}^{s,a} = \sum_{l} F_l^{s,a} P_l(\cos\theta)$$

Quasi-Classical ($q \ll p_F$ and $\omega \ll \varepsilon_F$) Kinetic Equation: Dynamics and Transport

- The quasiparticle Hamiltonian, $\varepsilon^{(0)}_{p\sigma}$, generates the quasi-classical equations of motion from using the Poisson Bracket, e.g.,

$$\frac{dn_{p\sigma}}{dt} = \frac{\partial n_{p\sigma}}{\partial t} - \{\varepsilon^{(0)}_{p\sigma}, n_{p\sigma}\}$$

- where,

$$\varepsilon^{(0)}_{p\sigma} = \varepsilon_{p\sigma} + \sum_{p'\sigma'} f^{\sigma\sigma'}_{pp'} \delta n_{p'\sigma'}$$

- Conservation laws follow from this equation, e.g. conservation of number density, n(r,t).

Linearized Quasi-Classical ($q \ll p_F$ and $\omega \ll \varepsilon_F$) Kinetic Equation

For small fluctuations in the quasi-particle distribution function, $n_p^{(0)}$, we have:

$$\frac{\partial n_p^{(0)}}{\partial t} + \vec{v}_p \bullet \vec{\nabla}\left(n_p^{(0)} - \frac{\partial n_p^{(0)}}{\partial \varepsilon_p^{(0)}} \delta\varepsilon_p^{(0)}\right) = I(n_p^{(0)})$$

Where,

$$\delta\varepsilon_p^{(0)} = \Phi_p^{ext} + 2\sum_{p'} f_{pp'}^{s} \delta n_{p'}$$

$$f_{pp'}^{s} = \sum_{l=0}^{\infty} f_l^{s} P_l(\cos\theta_{pp'})$$

and,

$$I[n_p] \propto T^2 \Rightarrow 0$$

Equations of Motion From the Kinetic Equation

The continuity equation (number is conserved):

$$\frac{\partial}{\partial t} n(r,t) + \frac{\partial}{\partial x_i} j_i(r,t) = 0$$

The equation of motion for the current (momentum is conserved): (T = 0)

$$\frac{\partial}{\partial t} j_i(r,t) + c_1^2 \frac{\partial}{\partial x_i} n(r,t) = 0$$

where,

$$c_1^2 = \frac{v_F^2}{3}(1+F_0^s)(1+\frac{F_1^s}{3})$$

"Hydrodynamic" Equation, T ≠ 0, Dynamics and Transport

For small fluctuations in the mass density, $\delta\rho(r,t) = \rho - \rho_0$ we have:

$$\frac{\partial^2 \rho(r,t)}{\partial t^2} - c_1^2 \nabla^2 \rho(r,t) = \frac{4}{3}\frac{\eta}{\rho_0}\frac{\partial}{\partial t}\nabla^2 \rho(r,t)$$

Where, $\eta = \frac{1}{5} n p_F v_F \tau_\eta$, $\rho(r,t) = mn(r,t)$

and, $v_F \tau_\eta = \lambda_\eta \propto T^{-2}$

In a 3D Fermi liquid the transport lifetimes and the quasi-particle lifetime have similar temperature dependence:

$$\tau_\eta(\tau_{(0)}) \propto \frac{1}{\left[|A|^2\right] T^2}$$

Spin Density and Current Equations of Motion

Spin density continuity equation (for B = 0):

$$\frac{\partial}{\partial t} m(r,t) + \frac{\partial}{\partial x_i} j_{\sigma,i}(r,t) = 0$$

Spin current equation, for paramagnetic Fermi liquid (B $\neq$ 0) :

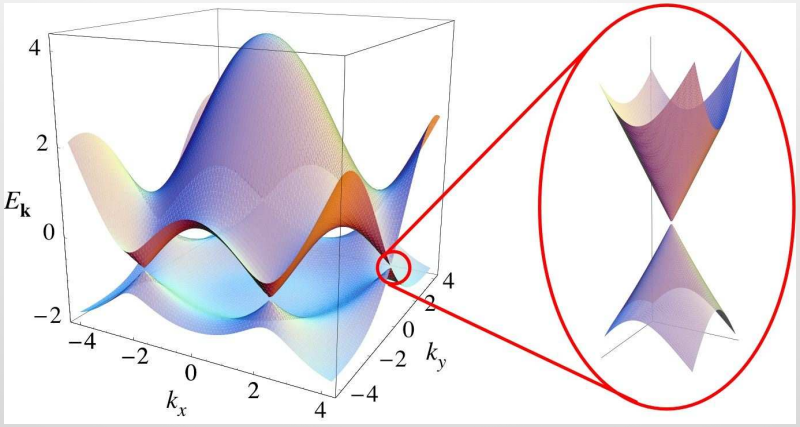
$$\frac{\partial}{\partial t} j_{\sigma,i}(r,t) + c_s^2 \frac{\partial}{\partial x_i} m(r,t) = \frac{-(2/\hbar)}{N(0)} (F_0^a - F_1^a/3) j_{\sigma,i}(r,t) \times m_0$$

where, for a paramagnetic Fermi liquid we have, $m_0 = \chi B$, and

$$c_s^2 = \frac{v_F^2}{3}(1+F_0^a)(1+\frac{F_1^a}{3})$$

Is Graphene an "Ultra-Relativistic" Metal?

- The dispersion is linear for some range in k-space:



Castro Neto

- Near K, K'

$$E_{\pm}(\mathbf{k}) \approx \pm\sqrt{3}\pi\gamma_0 a|\mathbf{k}|$$

$$v_f = \sqrt{3}\pi\gamma_0 a \approx 1 \times 10^6 m/s$$

Virial Theorem for Ultra-Relativistic Dirac Materials

Stokes, et al. Phil. Mag. Letters, 93, 672 (2013)

The Virial:

$$G = \sum_i \mathbf{r}_i \cdot \mathbf{p}_i$$

$$\dot{G} = \frac{1}{i\hbar}[G, H] = 0$$

$$= \frac{1}{i\hbar}\left(\sum_i [\mathbf{r}_i, T] \cdot \mathbf{p}_i + \sum_i \mathbf{r}_i \cdot [\mathbf{p}_i, V]\right)$$

$$= \sum_i \mathbf{p}_i \cdot \frac{\partial T}{\partial \mathbf{p}_i} - \sum_i \mathbf{r}_i \cdot \frac{\partial V}{\partial \mathbf{r}_i}$$

$$\sum_i \mathbf{p}_i \cdot \frac{\partial T}{\partial \mathbf{p}_i} = T \qquad \sum_i \mathbf{r}_i \cdot \frac{\partial V}{\partial \mathbf{r}_i} = -V - r_s \frac{\partial V}{\partial r_s}$$

Adding this all together:

$$\bar{T} + \bar{V} = -r_s \frac{\partial \bar{V}}{\partial r_s}$$

$$\bar{E} = -r_s \frac{\partial \bar{E}}{\partial r_s}$$

$$\bar{E} = \frac{\mathcal{B}}{r_s}$$

Recall: $\langle [G, H] \rangle = 0$

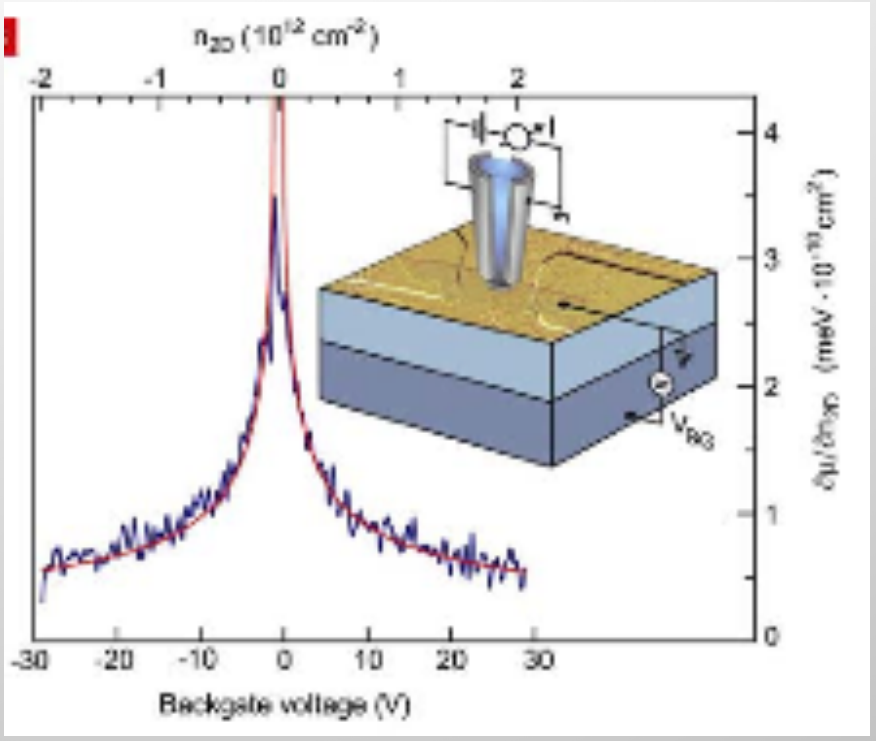
Results for Chemical Potential and the Sound Velocity from the Virial Theorem

$$P\Omega = -\frac{r_s}{d}\frac{\partial E}{\partial r_s} = \frac{E}{d}$$

$$\mu = (1+\frac{1}{d})\frac{\mathcal{B}}{N r_s}$$

$$\frac{1}{\bar{\kappa}} = \frac{\partial^2 \mathcal{E}}{\partial n^2} = \frac{\mathcal{B} a_0}{N}\frac{3}{4}\sqrt{\frac{\pi}{n}}$$

$$c_1^2 = \frac{1}{2}(v_f^0)^2(\frac{1+F_0^S}{1+F_1^S/2})$$



Martin, Nat. Phys. 4, 144 (2008)

v_f^0 and c_1 are independent of the density, thus, $\frac{1+F_0^S}{1+F_1^S/2}$, is as well.

Local Fermi Liquid for 3D Dirac Metals

Self Energy is momentum independent; (LFL) Theory, E&B, PRL '95 (Connection to DMFT).

$$\boldsymbol{\Sigma}(\mathrm{p}, \boldsymbol{\varepsilon}) \equiv \boldsymbol{\Sigma}(\boldsymbol{\varepsilon})$$

Local Landau interactions:

$$f_{pp'}^{\sigma\sigma'} = f_0^s + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' f_0^a$$

Local scattering amplitudes:

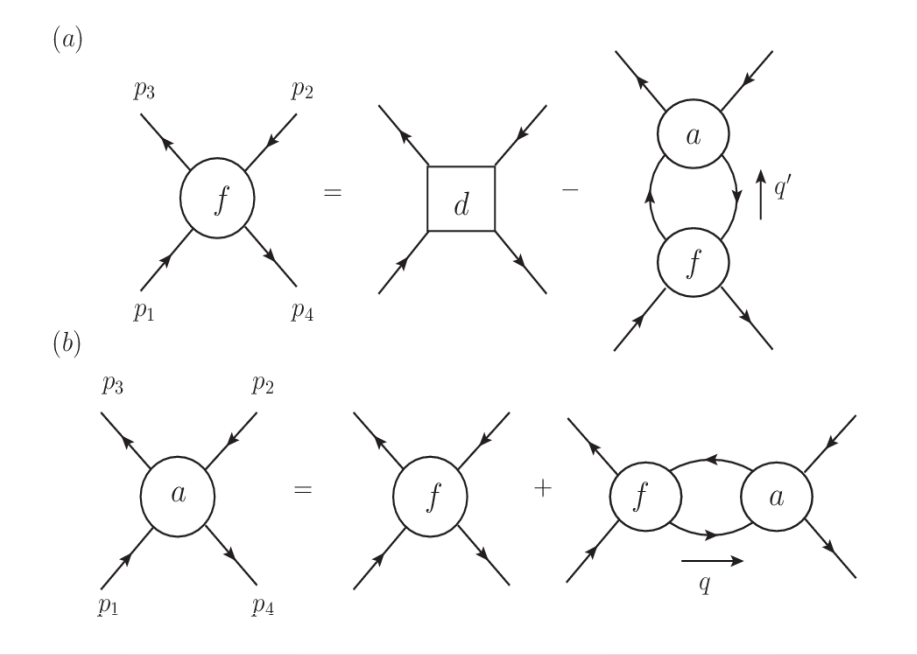
$$A_0^{s,a} = N(0)\, a_0^{s,a} = \frac{F_0^{s,a}}{1+F_0^{s,a}}, \quad \text{Pauli: } A_0^{\uparrow\uparrow} = A_0^s + A_0^a = 0$$

$$\textbf{Density of State}, \qquad N(0) = \frac{p_F^2}{\pi^2 \hbar^3 v_D}$$

Only 2 parameters, the Dirac velocity, v_f^0 , and f_0^s .

For Local Lorentz invariant systems, $f_l^{s,a} = 0, \mathbf{for}\ \mathbf{l} \geq \mathbf{1}$, only 1 parameter to fit. This is a long way from ∞ !

Induced Interactions for a Local Dirac Fermi Liquid



The Legendre polynomial moments, d_l, of the effective potential, $d(p_1,p_2:p_3,p_4)$, for the ultra-relativistic electron gas, are all 0, with the exception of d_0.* Including the spin dependence,

$$d_0^{\sigma\sigma''} = d_0^s + d_0^s\, \sigma \cdot \sigma'$$

** G. Baym and S. Chin, Nuclear Physics 1976*

Some consequences of the Local Dirac Liquid

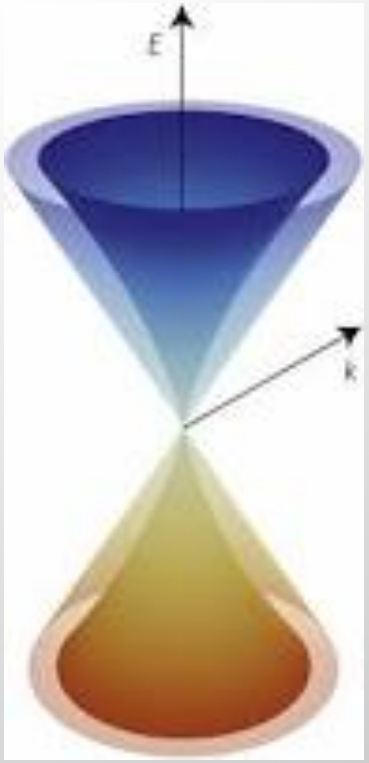
Are Dirac Cones Reshaped by Interactions?

If the Dirac liquid is local this would lead to a constraint on the ratio,

$$\frac{v_f^0}{v_F} = 1 + {F_1^s}/{3}$$

For F_1^s = 0, there would be no change in the Fermi velocity as a function of doping.

The outer part of the cone is the zero doping case and the inner one is the doped velocity.



Nature Phys. Lett
D. Elias, et al. 2011.

Consequences Continued

- First sound changes,

$$c_1^2 = \frac{1}{2}(v_f^0)^2(1 + F_0^s)$$

- Plasmon frequency changes,

$$\omega_{pl}^2 = 4\pi e^2 \frac{n}{m} \sim n, \text{ classical.}$$

$$\omega_{pl}^2 = 4\pi e^2 \frac{n}{p_F} v_f^0 \sim n^{2/3}, \text{ ultra-relativistic}$$

Summary and future Directions

- More calculations from the Fermi/Dirac Liquid. Response functions, transport, spin waves, etc.
- What if v_f^0 = c (the speed of light)?
- There would be bounds on the Fermi velocity, v_F , speed of sound, and zero sound would be bounded by c, putting restrictions on the Fermi liquid parameters.