Anomalous Thermalization in Non-Integrable Systems: Rare States and Lack of Heating Under a Drive

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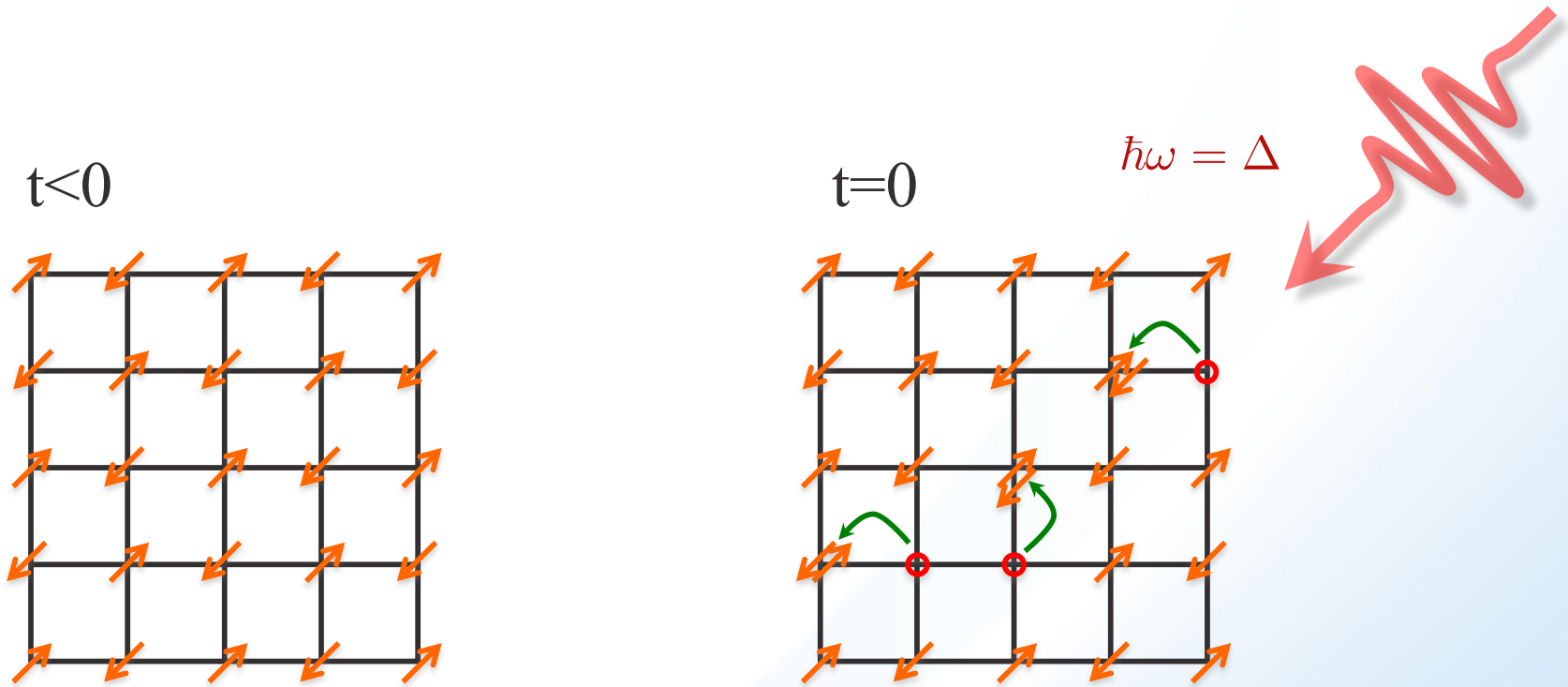
Thermalization in Closed Quantum Systems

I'm interested in the general problem of thermalization in a closed quantum system. That is, I imagine pumping energy into a system:



and then ask what is its subsequent long time behavior. And when I say the system is closed, I am imagining that it is coming to equilibrium via interactions with an outside world (there is no bath).

Long Time Behaviour

Observable $\langle \mathcal{O}(t) \rangle$

But what does equilibrated mean? Can we use notions from standard statistical mechanics to describe this late time value?

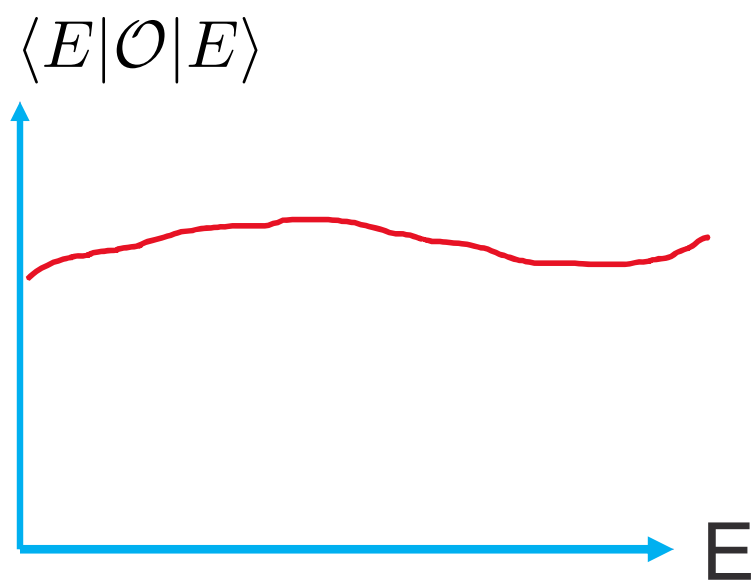
Long Time Behaviour and Eigenstate Thermalization Hypothesis

The answer is yes. We can relate the long time behaviour to the **microcanonical ensemble** via a conjecture known as the *Eigenstate Thermalization Hypothesis*.

This hypothesis, put forth by Mark Srednicki and David Deutsch, postulates that physical observables, $\mathcal{O}$, when evaluated on an eigenstate of the system's Hamiltonian of energy E, $|E\rangle$,

$$\langle E|\mathcal{O}|E\rangle$$

is a smooth function of E.



Exceptions to the Eigenstate Thermalization Hypothesis

But there are exceptions to the eigenstate thermalization hypothesis. And this is really what my talk is about:

There is one exception that I won't be talking about:

The system has a set of additional conserved quantities beyond energy, i.e. the system is integrable.

Now this can be a very interesting subject in and of itself. But I am going to be interested in the case where

$$\langle E|\mathcal{O}|E\rangle$$

is not a smooth function. States that violate this hypothesis are known as **rare states**.

Periodic Driving: Other Exceptions to ETH (?)

ETH has wider consequences than governing the long time behaviour after a quantum quench. It also has consequences for systems under periodic driving (with period, T).

We know from the Floquet theorem that at stroboscopic times, t=nT, the time evolution operator can be written in terms of a Floquet Hamiltonian, H_F:

$$U(t = nT) = e^{inTH_F}$$

If the spectrum of H_F satisfies ETH, we (generically) expect the system to heat to infinite temperature (D'Alessio and Rigol – PRX 4, 041048).

We are going to consider a possible exception to this rule in the periodic driving of a 1D Bose gas.

Outline

1. Eigenstate Thermalization Hypothesis (ETH) and Rare States

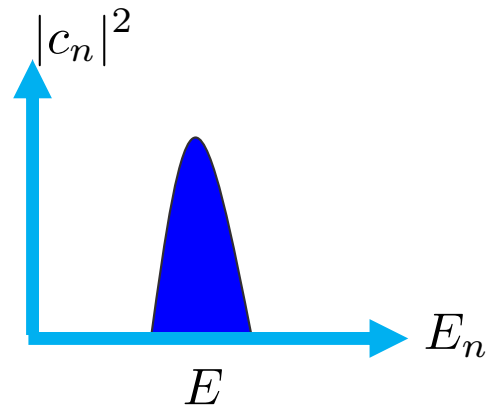
2. Rare States in 1D Quantum Ising
   - Bound states in 1D quantum Ising: Anomalous Thermalization
   - Bound states as rare states

3. Dynamical Localization in Periodically Driven 1D Bose Gases

Eigenstate Thermalization Hypothesis and Rare States

Eigenstate Thermalization Hypothesis

We imagine our system is initially in a state $|\psi\rangle = \sum c_n |n\rangle$ where the $|n\rangle$ are eigenstates of a Hamiltonian H with energies distributed narrowly about some energy E.



Suppose further that we have an observable $\mathcal{O}$ and ask what its long time value is, i.e.

$$\bar{\mathcal{O}} \equiv \lim_{t\to\infty} \langle\psi(t)|\mathcal{O}|\psi(t)\rangle \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \sum_{nm} e^{i(E_n-E_m)t} c_n^* c_m \langle n|\mathcal{O}|m\rangle = \sum_n |c_n|^2 \langle n|\mathcal{O}|n\rangle$$

We can now ask the question whether $\bar{\mathcal{O}}$ is thermal, i.e. whether it is equal to its microcanonical average $\mathcal{O}_{mc}$:

$$\bar{\mathcal{O}} \stackrel{?}{=} \mathcal{O}_{mc} \equiv \frac{1}{\mathcal{N}} \sum_{E-\Delta<E_n<E+\Delta} \langle n|\mathcal{O}|n\rangle$$

$\mathcal{N}$: no of states of states in energy window $E-\Delta < E_n < E+\Delta$

Eigenstate Thermalization Hypothesis

ETH asserts then that

$$\langle n|\mathcal{O}|n\rangle = A \quad \text{+ small corrections}$$

for all n in the microcanonical energy window $E-\Delta < E_n < E+\Delta$.

If the hypothesis holds, we automatically have thermalization:

$$\mathcal{O}_{mc} = \frac{1}{\mathcal{N}} \sum_{E-\Delta<E_n<E+\Delta} \langle n|\mathcal{O}|n\rangle = \frac{1}{\mathcal{N}} \sum_{E-\Delta<E_n<E+\Delta} A = A$$

$$\bar{\mathcal{O}} = \sum_n |c_n|^2 \langle n|\mathcal{O}|n\rangle = \sum_n |c_n|^2 A = A$$

But do we in fact have $\langle n|\mathcal{O}|n\rangle = A$ for all states?

Eigenstate Thermalization Hypothesis

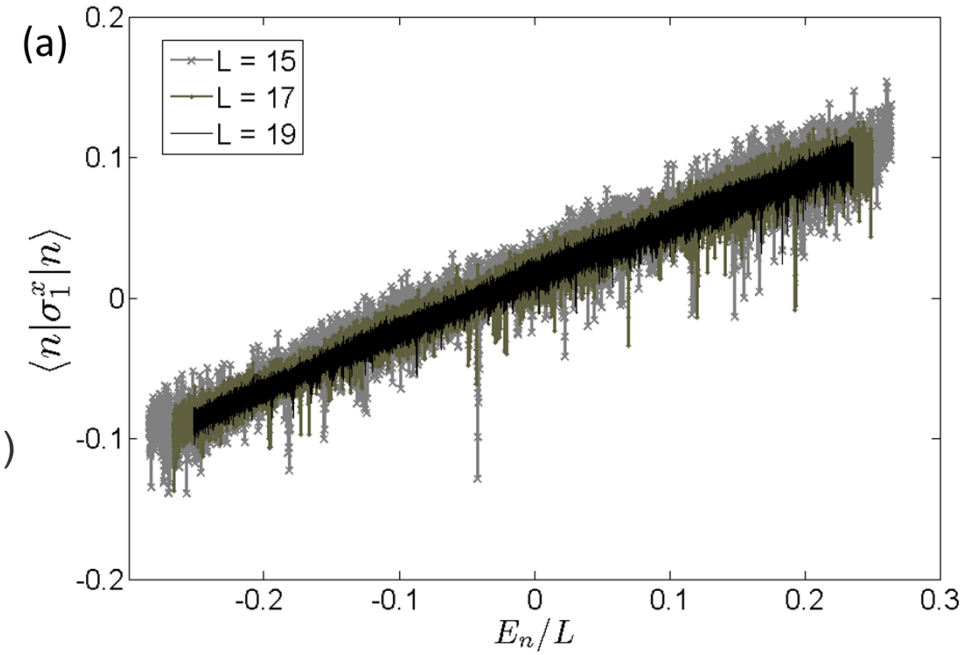
What should the expectation values (EVs) $\langle n|\mathcal{O}|n\rangle$ look like in a model where ETH holds?

1D quantum Ising:

$$H = \sum_i J\sigma_i^z\sigma_{i+1}^z + g\sigma_i^x + h\sigma_i^z$$

J=1, g=0.9045, h=0.8090

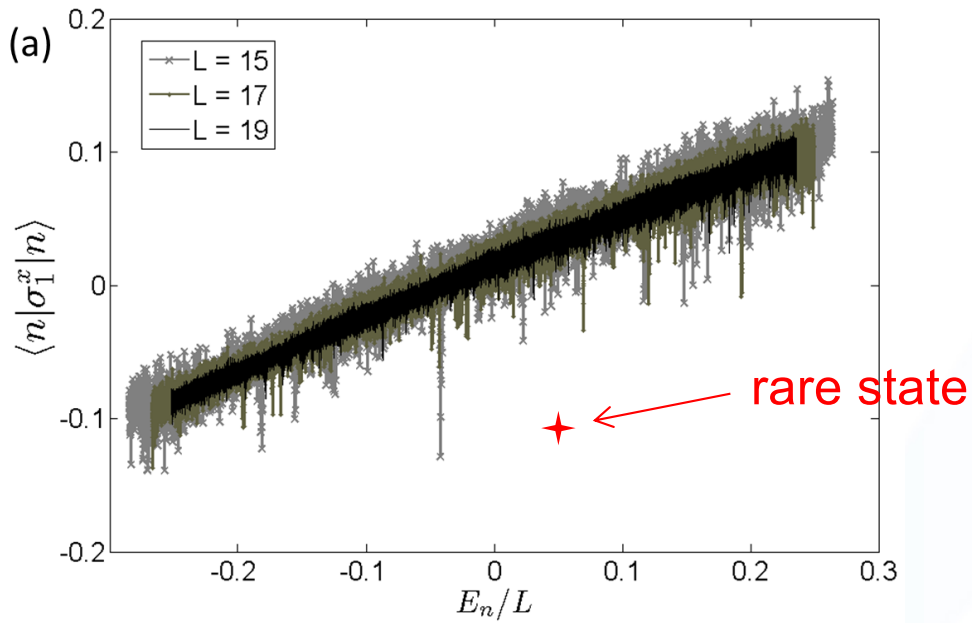
Kim, Ikeda, Huse, PRE 90, 052105 (2014)



For this choice of parameters, the EVs conform to ETH predictions: as the system size L gets larger, the deviation of EVs from their mean at a given energy shrinks.

Rare States

But what would happen if there was an outlier state, a state whose EV did not approach the mean with increasing system size, i.e.



Such a state is know as a “rare” state. If it were to exist, it would represent a violation of “strong” ETH. It is possible for rare states to exist but still have

$$\bar{\mathcal{O}} = \mathcal{O}_{mc}$$

In this case one says that “weak” ETH still is valid.

Examples of Rare States: Quantum Scars in Rydberg Atom Chains

In Rydberg atom platforms, there appear, apparently, ergodicity breaking states, so called quantum scars:

doi:10.1038/nature24622

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien[1], Sylvain Schwartz[1,2], Alexander Keesling[1], Harry Levine[1], Ahmed Omran[1], Hannes Pichler[1,3], Soonwon Choi[1], Alexander S. Zibrov[1], Manuel Endres[4], Markus Greiner[1], Vladan Vuletic[2] & Mikhail D. Lukin[1]

The Rydberg atom (two-level) Hamiltonian under a laser drive is given by:

$$H = \sum_{j=1}^{L} \left(\frac{\Omega}{2} X_j - \Delta Q_i \right) + \sum_{i<j}^{L} V_{i,j} Q_i Q_j$$

X_j is Pauli matrix σ_x on site j
$Q_j = (1 + Z_j)/2$

We are interested in the limit that $V \gg \Omega$. In this limit our Hamiltonian becomes very simple:

$$H = \sum_{j=1}^{L} P_{j-1} X_j P_{j+1}$$

$P_j = (1 - Z_j)/2$

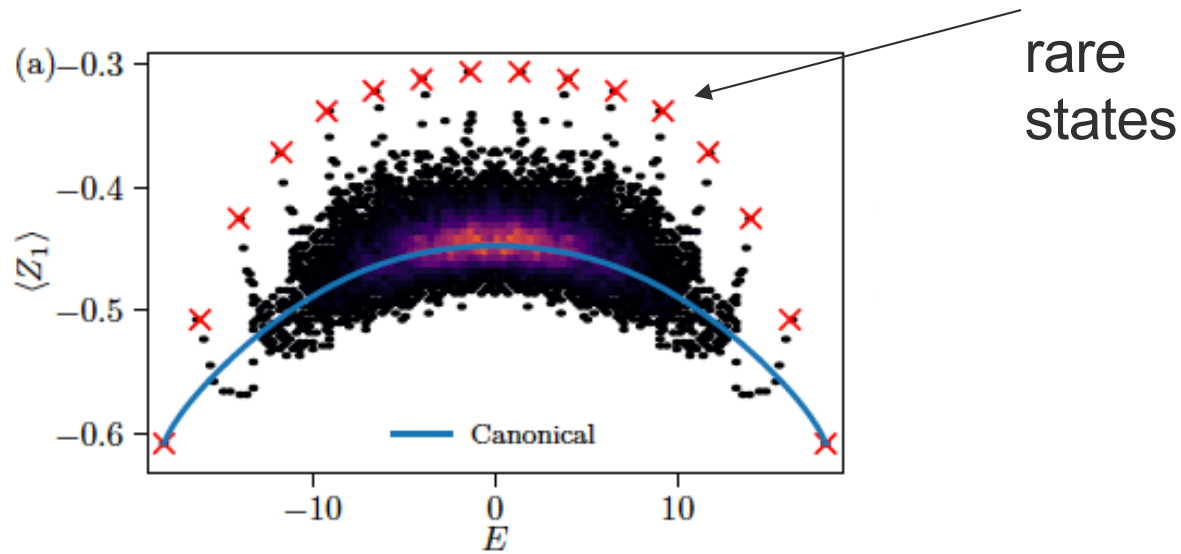
It has an effective constraint that no two excitations (spin ups) can be side by side.

Examples of Rare States: Quantum Scars in Rydberg Atom Chains

This model appears to support rare states (Turner, Michailidis, Abanin, Serbyn, Papic, Nat. Phys. 14, 745 (2018)):

Expectation values of operator

$$Z_1 = \sum_j Z_j$$



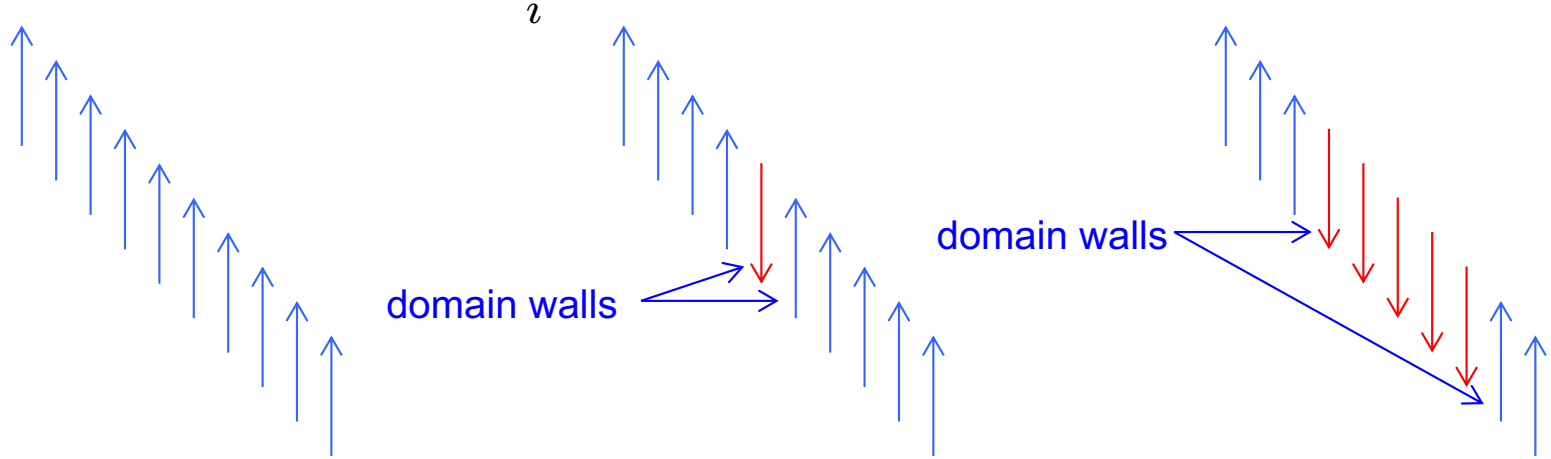
The rare states appear to be CDW states, i.e. $|\mathbb{Z}_k\rangle = |\ldots \underbrace{\bullet\circ\ldots\circ}_{k}\bullet\ldots\rangle$

Bound States in 1D Quantum Ising in a Longitudinal Field

Linearly confined bound states

$$H = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z + g\sigma_i^x + h\sigma_i^z \right)$$

h=0

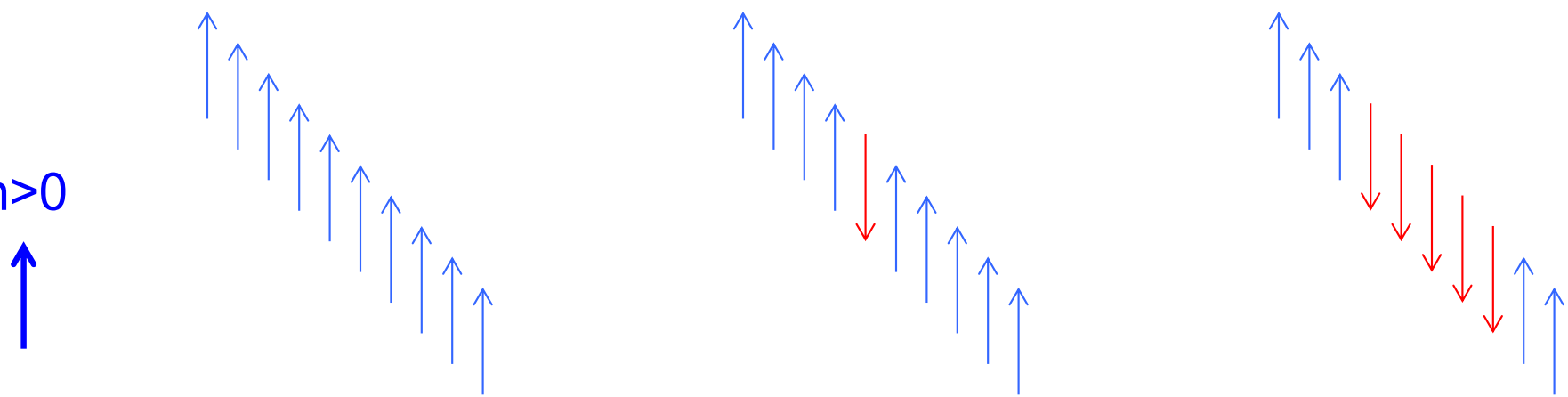


ordered ground state

excited state with two domain walls

with h=0, domain wall are free to move apart

h>0



ordered ground state

excited state with two domain walls

with h>0, a distance d between domain walls costs hd in energy

Consequences of Bound States in 1D Quantum Ising

Entanglement growth and correlation propagation post-quench

How do entanglement and correlations spread in a system after a quantum quench?

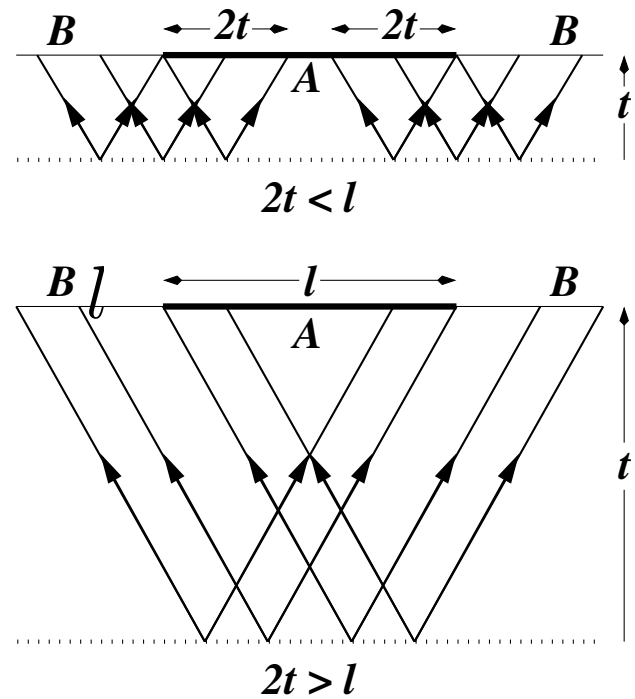
There are two rules: (Cardy+Calabrese J. Stat. Mech, 2005; PRL 2006)

Rule 1, Horizon/lightcone effect: Two-point functions

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle$$

become stationary for times

$$t > |x_1 - x_2|/2v$$



quasi-particle propagation post-quench

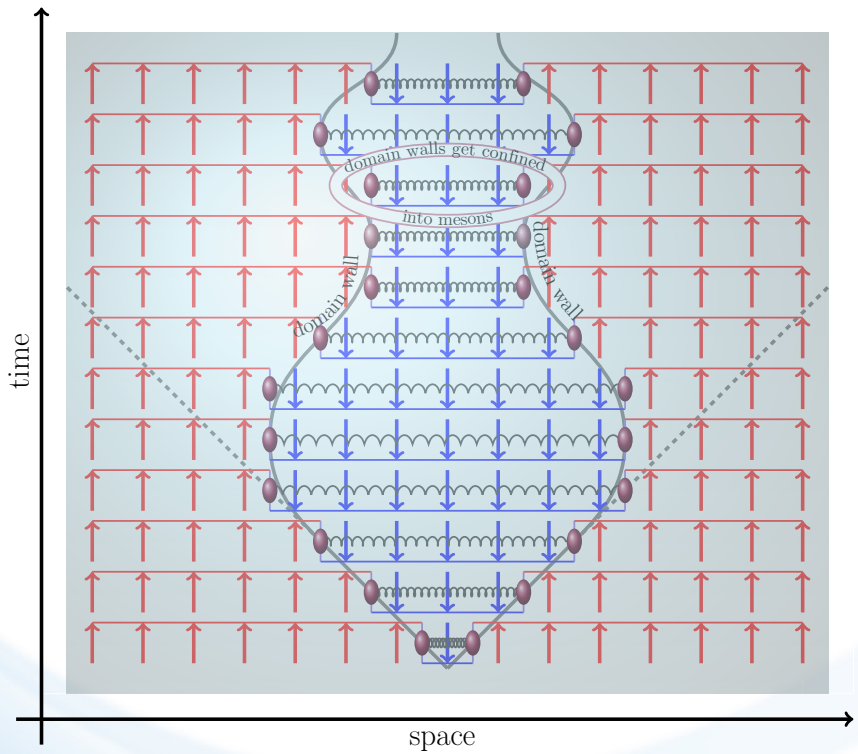
Rule 2, Linear growth in time of S_{EE}: For an interval A of length l, S_{EE} reaches a saturation after $l/2v$ to a value of $\sim l/\beta$.

Entanglement entropy growth and propagation post-quench

This basic intuition breaks down in the absence of simple freely propagating quasi-particles, i.e. confined bound states such as found in the quantum Ising model:

$$H = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z + g\sigma_i^x + h\sigma_i^z \right)$$

quantum Ising model in a longitudinal field



The domain walls created by the quench in the 1D quantum Ising model do not fly apart because of the confining longitudinal field. And so the growth of entanglement entropy is not linear in time.

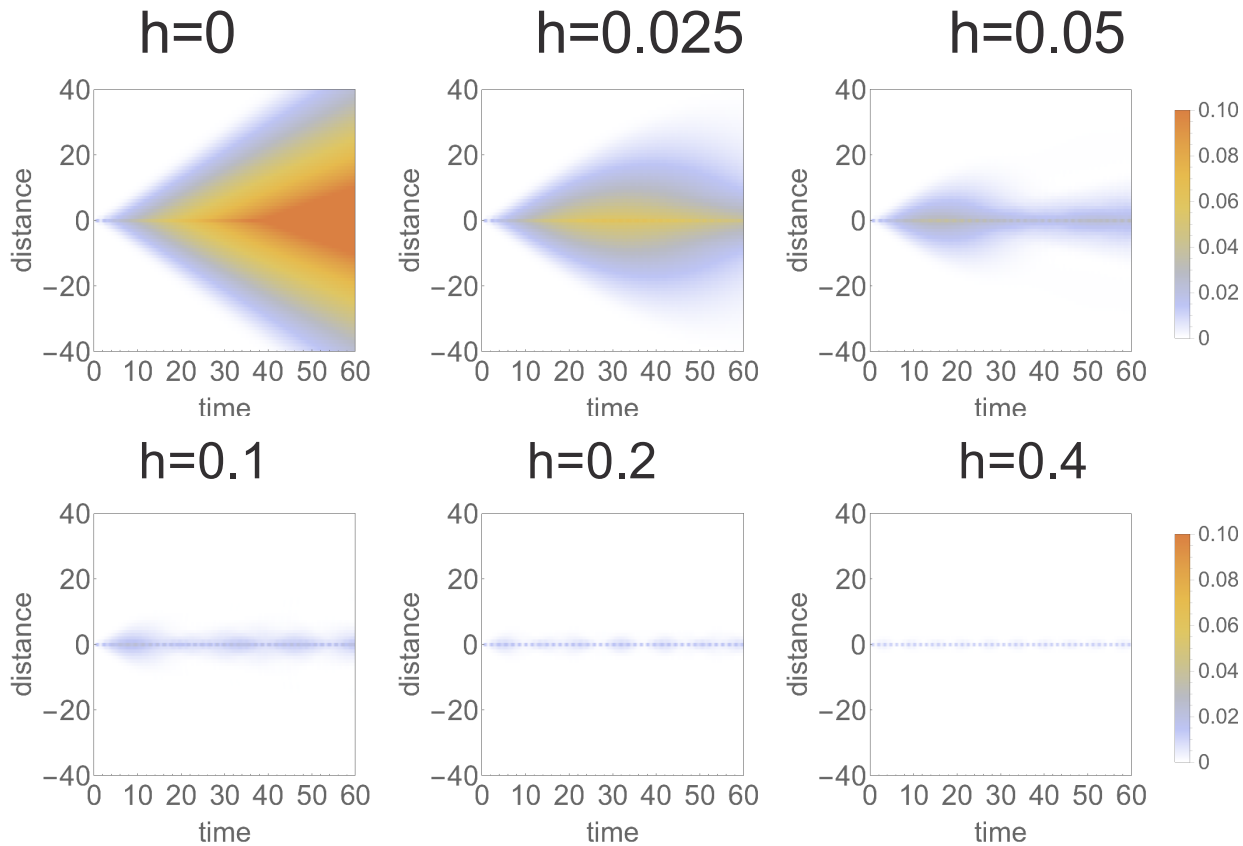
Kormos, Collura, Takács, Calabrese: Nature Physics (2016)

Bound states and anomalous post-quench behaviour

Consider a quench:

$$H = -J\sum_i(\sigma_i^z\sigma_{i+1}^z + h\sigma_i^z) \xrightarrow{\text{t=0}} H = -J\sum_i(\sigma_i^z\sigma_{i+1}^z + 0.25\sigma_i^x + h\sigma_i^z)$$

Light cone plots for $\langle\sigma_1^z\sigma_{m+1}^z\rangle_c$



Kormos et al. (2016)

Bound states lead to anomalous thermalization. But are they rare?

Studying Bound States as Rare States via the Truncated Spectrum Approach

Truncated Spectrum Approach (TSA)

This approach is an approach for treating continuum theories. So we are not going to treat

$$H = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z + g\sigma_i^x + h\sigma_i^z \right)$$

but its continuum version instead, a theory of a massive relativistic Majorana fermion

$$H = \int \frac{dx}{8\pi} [iv(\bar{\psi}\partial_x\bar{\psi} - \psi\partial_x\psi) - 2im\bar{\psi}\psi + \tilde{h}\sigma] \quad \begin{array}{l} m \sim (1-g)J \\ v \sim Ja \\ \tilde{h} \sim \frac{Jh}{a^{7/8}} \end{array}$$

Here there is an important question of whether a lattice model and a continuum model should see the same thermalization patterns.

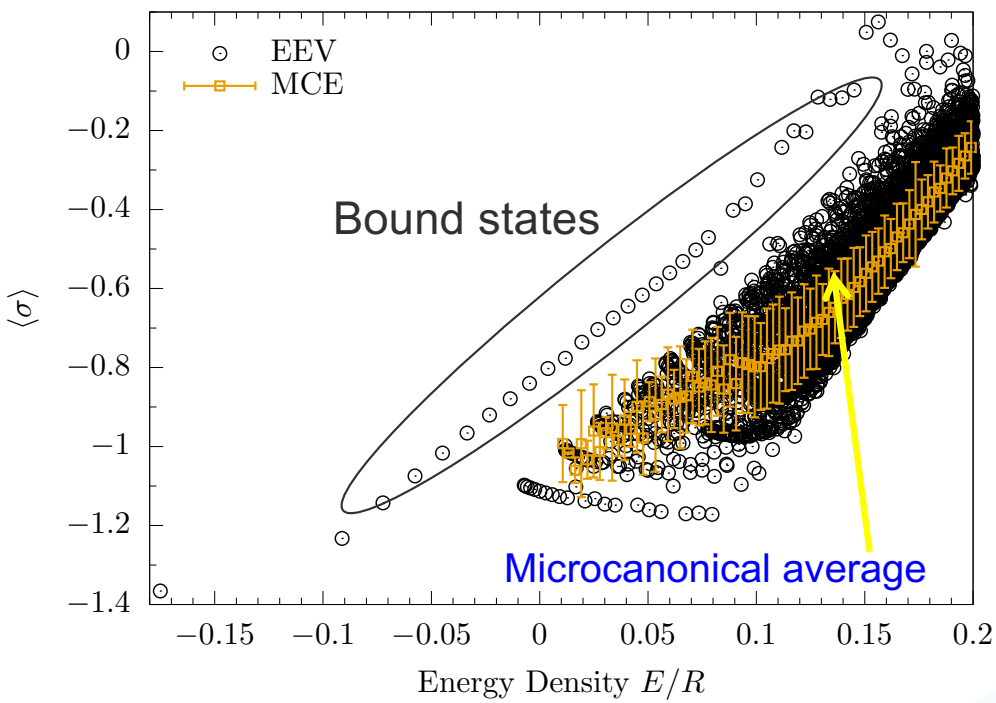
TSA: Methodology works by treating the field theory in finite volume and finite UV cutoff. Effects of the UV cutoff are ameliorated with RG strategies.

Review: A. James, RMK, P. Lecheminant, N. Robinson, A. Tsvelik, Reports in Progress in Physics 81 (2018)

Linearly confined bound states as rare states

Eigenstate Expectation Values for Ising

$$H^{1D\ Quantum\ Ising} = \int \frac{dx}{8\pi}[iv(\bar{\psi}\partial_x\bar{\psi} - \psi\partial_x\psi) - 2im\bar{\psi}\psi + h\sigma]$$



There exists a branch of states that do not match the microcanonical ensemble, i.e. rare states.

We know these states are bound states because we can solve a Bethe-Salpeter equation that gives us the wavefunction and energies of the bound states.

Does this survive into the thermodynamic limit? Ask me if curious.

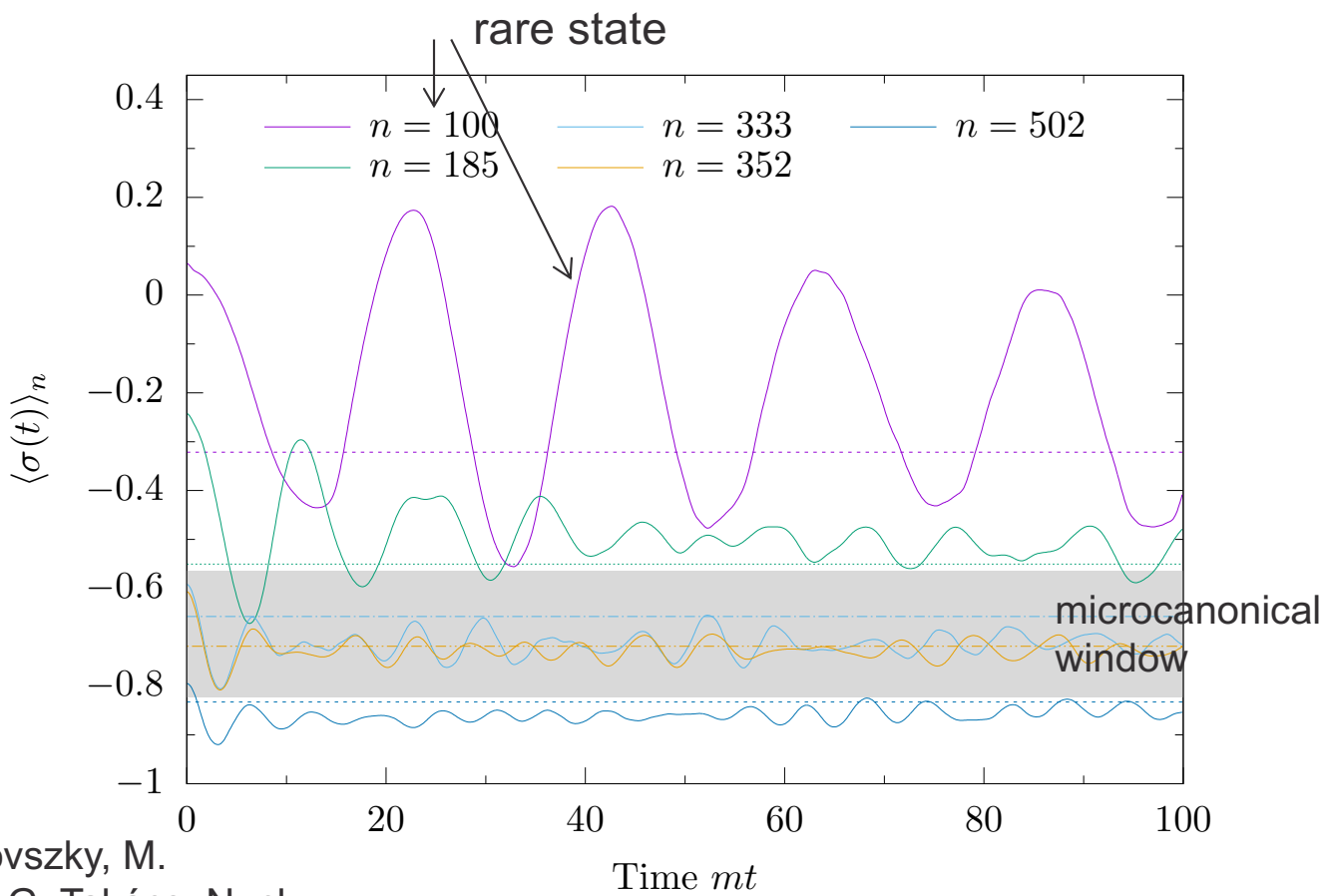
Can Rare States Control Post-Quench Thermalization

Or is their existence consistent with weak ETH?

If we initialize the system in a rare state for one value of h and then quench to a different value of, we find the expectation value does not equilibriate to the microcanonical value.

Time evolution following a quench from h=0.1 to h=0.2.

We choose as initial conditions different eigenstates of the h=0.1 Hamiltonian. State 100 here is a rare state.



Chebyshev methods for TSA: T. Rakovszky, M. Mestyán, M. Collura, M. Kormos, and G. Takács, Nucl. Phys. B 911, 805 (2016)

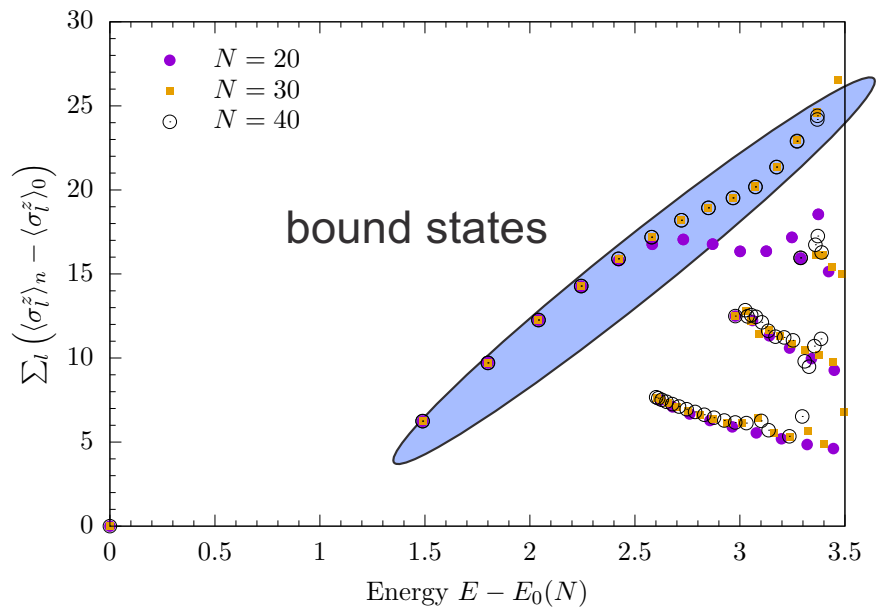
Are the Existence of Rare States a Feature of the Continuum?

Answer: No (probably).

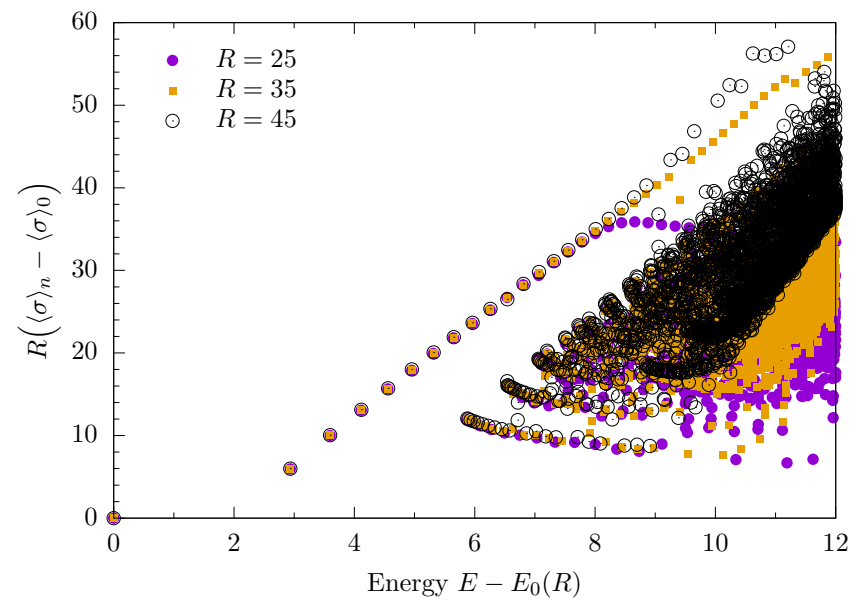
They seem to appear in the lattice model as well:

fDMRG computation on a 40-site lattice with first 200 states plotted.

OBCs are used – here bound states are boundary bound states involving a single fermion.



As a comparison, our continuum results.



Dynamical Many-body Localization

Basic Setup

1D Bose Gas Under a Periodic Drive:

$$H = \int_0^L dx \Big[\overbrace{-\frac{\hbar^2}{2m}\psi^\dagger(x)\partial^2\psi(x) + c\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x)}^{\text{Lieb-Liniger model}} + \overbrace{V\cos(qx)\psi^\dagger(x)\psi(x)\sum_{n=1}^{\infty}\delta(t-nT)}^{\text{Periodic kicking term}} \Big]$$

Primary Question: How does the system's energy evolve in time? Does it flow off to infinity or remain finite?

Challenge: We need to study the time evolution to very long times.

Single Particle Case

This same question has been asked in the single particle case: a quantum kicked rotor motor:

$$H = -\frac{h^2}{2I}\partial_\theta^2 + V(\theta)\sum_{n=1}^{\infty}\delta(t-nT)$$

Here by mapping the system onto a 1D disordered Anderson model, Fishman, Grempel, and Prange (PRL 49, 509 (1982)) showed that there is energy localization. This was later confirmed in experiments by Mark Raizen's group:

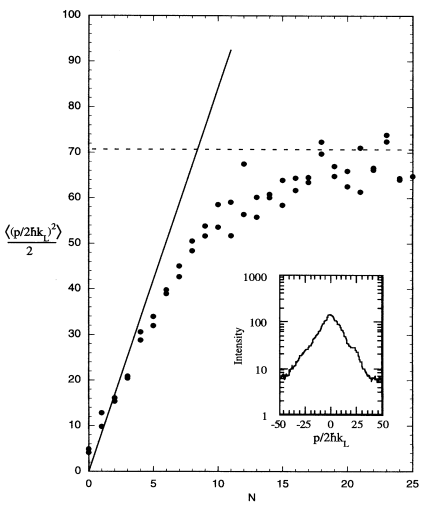


FIG. 4. Energy $\langle (p/2\hbar k_L)^2\rangle/2$ as a function of time. The solid dots are the experimental results. The solid line shows the calculated linear growth proportional to the classical diffusion constant $\kappa^2/2$. The dashed line is the saturation value computed from the theoretical localization length ξ. The inset shows an experimentally measured exponential line shape on a logarithmic scale which is consistent with the prediction $\xi = \kappa^2/4k^2 \approx 8.3$.

They saw energy saturation after 10 kicks (short quantum break time).

Moore et al
PRL 75, 4598 (1995)

This means we will see localization for c=0 and c=∞.

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Generalized Hydrodynamics

Generalized hydrodynamics (GHD) is a coarse grained description of a quantum integrable system where it is assumed that a local equilibrium exists that is described not only by energy conservation but conservation of all conserved quantities of the integrable model.

The GHD equation for the gas in the presence of an external force is:

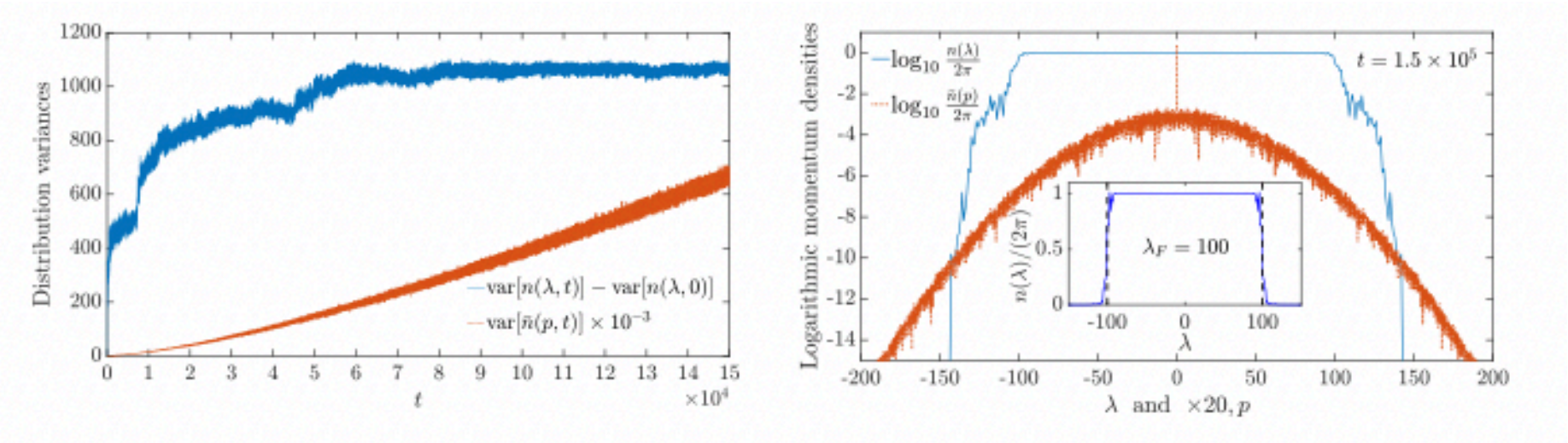
$$\partial_t n(x,t,\lambda) + \partial_x(v_{eff}(x,t,\lambda)n(x,t,\lambda)) = \frac{\partial_x V(x,t)}{m}\partial_\lambda n(x,t,\lambda)$$

Doyon, Yoshimura, SciPost Phys. 2, 014 (2017)

Here $n(x,t,\lambda)$ is the density of excitations with quasi-momentum λ.

It is a much easier equation to solve than the fully quantum dynamics, and it gives one a chance to propagate the system to late times *and* work with large system sizes.

Results



At large (but finite c), we see that the distribution (blue) of quasi-momenta localize. However the Fourier transform of the spatial density (orange) does not localize within the times that we are able to simulate.

The wave vector q and V chosen so that many-body effects are maximized and the chance of localization due to effects present in the classical limit (the Chirikov standard map) are minimized.

We see (at least) that dynamical localization occurs over very long time scales.

Summary

1. We have argued that there are “rare” states in 1D quantum Ising.

   - Such rare states have been recently discovered in kinetically constrained models where they have been termed quantum scars.

   - They are also present in 2D quantum Ising. Seeing them required combining TSA and MPS approaches (ask me if curious).

2. We have demonstrated that energy localization in a periodically driven Lieb-Liniger model occurs at least for large c and long times.